

Summary of the manuscript “On the possibilities of predicting geomagnetic secular variation with geodynamo modeling” to be submitted to Geophysical Research Letter (GRL)

It has long been known that the Earth possesses an internal magnetic field (i.e. the field vanishes at infinite distance from the Earth). This geomagnetic field is believed to be generated and maintained by convective flow in the Earth’s liquid outer core (geodynamo). In the past decade, several numerical models have been developed to model the geodynamo, including our MoSST (Modular, Scalable, Self-consistent and Three-dimensional) core dynamics model. These models can successfully explain qualitatively the geodynamo process in the outer core, e.g. a dominantly dipolar geomagnetic field at the surface (with the polarity almost parallel to the geometric polarity of the Earth), westward-drift of geomagnetic field lines and reversals of geomagnetic polarity.

However, no attempt has been given to quantitative applications of the numerical models on geomagnetic field, partly due to large differences between the parameters used in numerical simulations and those appropriate for the Earth’s core, and partly due to physical approximations adopted in numerical modeling. But such studies are important for geodynamo and geomagnetic research: surface geomagnetic observations can be used to constrain numerical geodynamo models, and numerical models can be applied to forecast geomagnetic secular variation observable on and near the Earth’s surface.

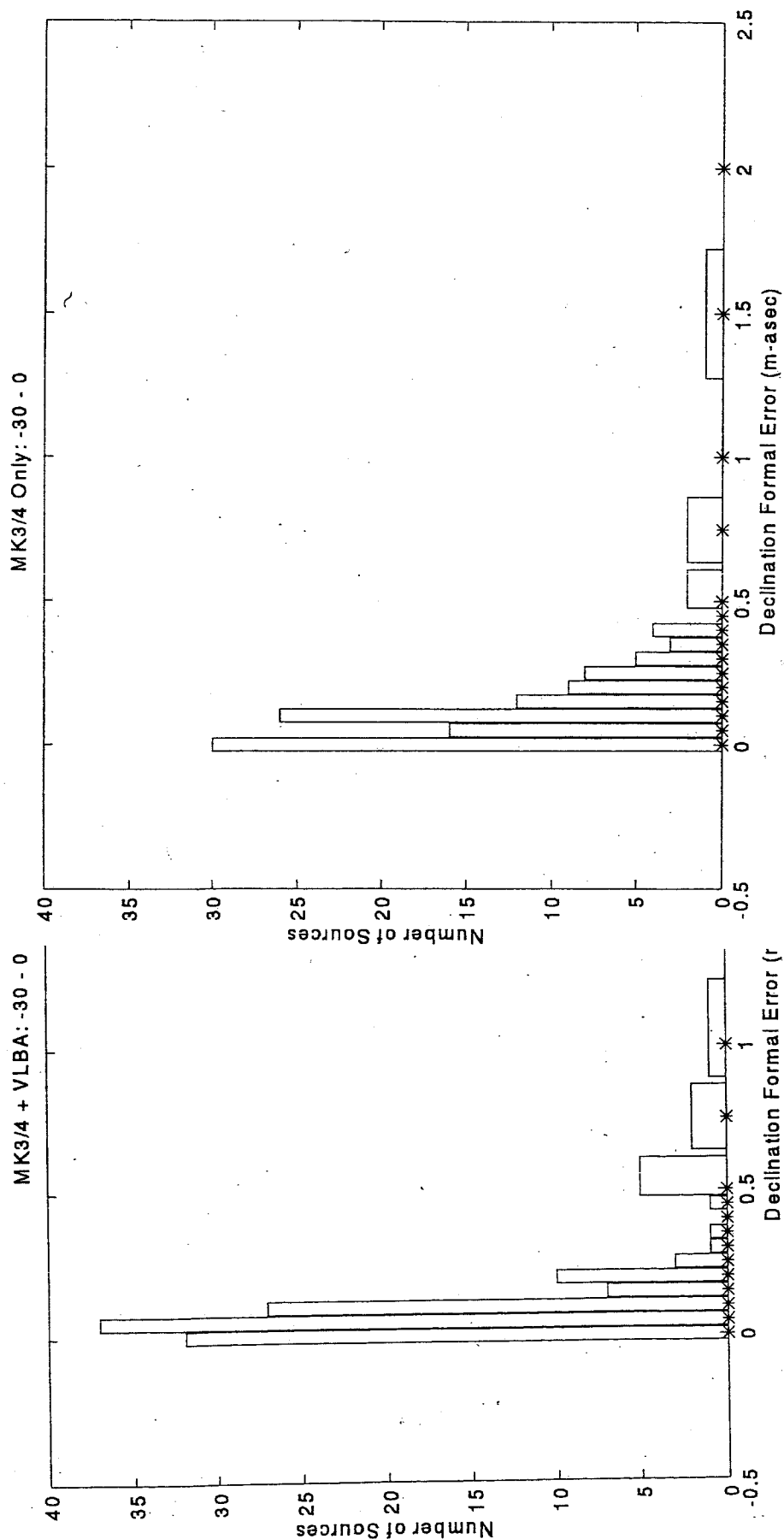
This research article reports our first ever effort on applying our MoSST core dynamics model and the observed surface geomagnetic field to predict geomagnetic secular variation. The surface geomagnetic field is obtained via the comprehensive field model operated here in GSFC. As the first attempt, we focus on examining how numerical dynamo solutions are affected by geomagnetic observation. For this purpose, a pure numerical geodynamo solution is selected to be assimilated with the surface geomagnetic field in 1940. The assimilation is simple: the observed field is inserted into the dynamo solution. The new solution is then used as an initial state for numerical simulation. The simulated solutions are then used to compare with the observed surface geomagnetic field in the subsequent years.

Our findings are very encouraging. While there is no correlation between the field from pure dynamo simulation and the field from observation, the field from the new solutions with data assimilation, in particular the large-scale (or low degree) field evolves closely with the observed field over time. As the result, the assimilated field can capture large-scale features, such as south Atlantic anomaly observed at the surface of the Earth for the period from 1940 to 1990. However, there are still discrepancies between the small-scale (high degree) field. In addition, the assimilated solutions diverge (without further assimilation constraint) above approximately 60-year periods.

Our research results suggest that it is possible to assimilate numerical results with surface observations to predict future changes in geomagnetic field. They also suggest that further research is necessary on better understanding the statistical properties of numerical dynamo solutions, in particular the error development in time, and the stability of the numerical solutions under arbitrary perturbations. These are necessary for future development/implementation of better assimilation technologies.

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Distribution of Source Formal Errors $0 \Rightarrow -30$ Declination



On the Possibilities of Predicting Geomagnetic Secular Variation with Geodynamo Modeling

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We use our MoSST core dynamics model and geomagnetic field at the core-mantle boundary (CMB) continued downward from surface observations to investigate possibilities of geomagnetic data assimilation, so that model results and current geomagnetic observations can be used to predict geomagnetic secular variation in future. As the first attempt, we apply data insertion technique to examine evolution of the model solution that is modified by geomagnetic input. Our study demonstrate that, with a single data insertion, large-scale poloidal magnetic field obtained from subsequent numerical simulation evolves similarly to the observed geomagnetic variation, regardless the initial choice of the model solution (so long it is a well developed numerical solution). The model solution diverges on the time scales on the order of 60 years, similar to the time scales of the torsional oscillations in the Earth's core. Our numerical test shows that geomagnetic data assimilation is promising with our MoSST model.

1. Introduction

Through much of its history, the Earth has possessed an internal magnetic field (geomagnetic field) that is believed generated and maintained by convective flow in the fluid outer core (geodynamo) [Larmor 1919, @]. However, it is only for less than a decade, that numerical models have been successfully developed to simulate self-consistent, fully nonlinear geodynamo processes [Glatzmaier and Roberts 1995, @; Kageyama and Sato 1997, @; Kuang and Blozham 1997, @]. These models, though different in many aspects (e.g. algorithms and physical approximations), are able to generate Earth-like magnetic field at the CMB. For example, numerical solutions show a dominant dipolar field at the CMB, large-scale westward drift, and occasional field polarity reversals [Konon and Roberts 2002, @].

However, geomagnetic and paleomagnetic observations are not directly utilized in numerical geodynamo modeling, mainly because the numerical parameter domains are far from that for the Earth's core. This handicaps our understandings on the geodynamo mechanisms, thus limiting model improvements and geophysical ap-

plications. For example, numerical models can produce different solutions that are similar at the CMB, but very different deep inside the outer core [Kuang and Bloxham 1997, @; Kuang 1999, @]. This is partly caused by different approximations in the models on torque balances on the co-axial (with the Earth's rotation axis) cylindrical surfaces across the outer core (the Taylor cylinders). Thus surface observations could help identifying appropriate approximations for geodynamo modeling.

Incorporating observations to numerical modeling can facilitate an important application: predicting geomagnetic secular variation via data assimilation. This is not new, as similar developments occurred in meteorology and oceanography, where large-scale circulation models are used together with past and current observations to predict changes in the future. More recently in solid Earth research, numerical mantle convection models are used together with current observations to hindcast historical mantle flow [Bunge et al 2003, @]. In these approaches, appropriate assimilation techniques are selected to enable us applying the known physics (the models) to understand observations (data), and using discrepancies among model outputs and observational data to improve physics knowledges. Similarly in geomagnetic data assimilation, observations could be used as "time stamps" to modify/constrain the numerical solutions, such that the modified solutions shall evolve closely following the "true" observational trend.

However, several limitations in geomagnetic observations could pose serious obstacles to the assimilation. First, only the poloidal part B_P of the core field B can be observed above the Earth's surface. The toroidal component B_T is filtered by a thick, poor electrically conducting mantle. Next, B_P is significantly attenuated by the crustal magnetic field, leaving only the large scale (for degree $L \leq 13$ in spherical harmonic expansion) signals observable above the surface. In addition, data record is short and quality decreases back in time: there are about 40 years of the highest quality data from global satellite measurements [Sabaka et al 2002, @]. Ground station and navigation observations provide less accurate records over the past centuries [Bloxham and Jackson 1991, @]. Poorer paleo/archeomagnetic records could extend the observed surface field distribution back to more than 3000 years [Constable et al 2000, @]. Combined the observation record is a fraction of the free-decay time scale τ_d (≈ 20000 years) in the Earth's core. Therefore the immediate question is whether a modified solution with one "time stamp" of geomagnetic observations assimilated to a numerical dynamo solution could evolve sufficiently close to observations within a reasonable time interval.

Kuang [2000] reported some initial tests on one "time stamp" assimilation. His solution suggested that the modified solution evolves following much closer to observations than purely dynamo simulation. However, no attempt was made to quantify the tests, in particular error development, which is very important for understanding the impact of assimilation processes to the dynamics in the core, and thus to the evolution of the geomagnetic field.

In this paper we repeat the tests. In particular we shall focus on the error development, and the differences between the assimilated solutions and the observations. The latter shall be used to measure the "improvement" of the numerical solutions compared with the observations.

2. Assimilation Algorithm

During the past thirty years, assimilation methods have evolved from simple insertion methods, in which observation values replace model outputs whenever available, to more sophisticated techniques that require detailed knowledge of error statistics. Since error statistics for geodynamo models are unknown, we consider here a simple assimilation technique similar to data insertion [Berry and Marshall 1989, ②]. Our main purpose is to examine the sensitivity of the numerical model to assimilation process and begin to obtain error information that can be used to improve the assimilation scheme.

This approach can be briefly described as follows. The magnetic field B in our model is:

$$B = B_T + B_P \equiv \nabla \times (T \mathbf{1}_r) + \nabla \times \nabla \times (P \mathbf{1}_r) \quad (1)$$

where $\mathbf{1}_r$ is the radial unit vector, T and P are the toroidal and poloidal scalars, respectively. The two scalars are expanded in spherical harmonics,

$$P = \sum_{0 \leq m \leq l}^{l \leq L} b_l^m(r, t) Y_l^m(\theta, \phi) + C.C., \quad (2)$$

where $\{Y_l^m\}$ are fully normalized spherical harmonic functions, $C.C.$ denotes the complex conjugate part, and (r, θ, ϕ) define the spherical coordinate. The poloidal scalar P is further divided into two parts

$$P = P_1 + P_2, \quad (3)$$

such that

$$P_1 = \sum_{0 \leq m \leq l}^{l \leq L_1} b_l^m Y_l^m + C.C., \quad (4)$$

where $L_1 < L$ denotes the truncation order in observations.

In the insertion, P_1 is first modified at the CMB $r = r_{\text{cmb}}$ via

$$(b_l^m/b_1^0)_{\text{assim}} = (b_l^m/b_1^0)_{\text{obs}} \equiv \alpha_l^m \text{ for } l \leq L_1. \quad (5)$$

It is then assumed broadcasted "instantly" to the entire core via

$$(b_l^m/b_1^0)(r \leq r_{\text{cmb}}) = \alpha_l^m. \quad (6)$$

Thus,

$$P_1^{\text{assim}} = b_1^0(r) \sum_{0 \leq m \leq l}^{l \leq L_1} \alpha_l^m Y_l^m + C.C. \quad (7)$$

Next, P_2 is modified as

$$P_2^{\text{assim}} = \beta P_2, \quad (8)$$

where the multiplier β is determined to conserve the poloidal field energy in the outer core

$$\int_{\text{core}} [B_P^{\text{assim}}]^2 dV = \int_{\text{core}} B_P^2 dV. \quad (9)$$

Other quantities, e.g. the toroidal scalar T and the ve-

locity field \mathbf{v} , are not changed.

The modification (7) implies that the insertion is defined relative to the dipole component b_1^0 . The approximation (6) intends to reduce the transient time for the insertion. The conservation of the energy ensures that the insertion is neither an energy source nor a sink.

3. Numerical Results

In our test here, an arbitrary dynamo solution is selected to assimilate with the observational data for the year 1940. The time evolution of the modified solution (the model output) is then used to compare with those post 1940 observations derived from the comprehensive model [Sabaka et al 2002, ①].

In our tests, the parameters are chosen as

$$R_o = E = 1.25 \times 10^{-6}, \quad p_\kappa = 1, \quad (10)$$

we refer the reader to Kuang and Bloxham [1999] for the definitions. We also choose $L_1 = 8$ in the insertion, which is slightly lower than the order $L_1 = 13$ of the core field observed at the Earth's surface.

In Figure 1, we plotted several spectral coefficients b_l^m/b_1^0 of the data insertion test (dashed lines) and of the original dynamo simulation (dotted lines) against the observations (the solid lines). From the figure we can observe that the time evolution of the field coefficients have been greatly affected by the insertion: some evolve closely to the observations, but some diverge rapidly from the observations. However, more coefficients follow the observations. In contrast, there is no positive correlation between purely dynamo simulation outputs with the observations. In fact, a simple error analysis demonstrated that the difference between the data insertion outputs and the observations are orders of magnitude smaller than that between the purely dynamo simulation outputs and the observations, as shown in Figure 2.

Since more coefficients evolve similarly with the observations, we expect that the radial component B_r of the magnetic field at the CMB from the model outputs should be similar to those inverted from surface geomagnetic observations, as shown in Figure 3. In particular, we can observe from the figure that the large scale features, such as the south-Atlantic anomaly, are well captured in the model outputs, consistent with the coefficients variations. However, we should notice that the small scale features, such as the magnetic spots in mid Atlantic region (e.g. Bermuda area) and in Pacific region are different. While some of these features are not consistently reproduced within geomagnetic field models, the others indicate that continuous assimilation is necessary for better field prediction.

The simulation with the one-step data insertion lasts approximately $0.1 \tau_d$ in modeling time (or equivalently 60 years). After that numerical solutions diverge quickly beyond the model resolution limit. Part of the explanation is that the delicate torque balance on the Taylor cylinders is significantly offset by the insertion (6) (see Figure 4), resulting in instabilities on the torsional oscillations (with the frequencies of order $0.1 \tau_d$) that are controlled by the torque balance [Braginsky 1976, ②].

4. Discussion

In this article we described our first attempt on geomagnetic data assimilation. Using a one-step data inser-

Figure 1

Figure 2

Figure 3

Figure 4

tion approach (5)-(9), we are able to modify numerical outputs such that at the CMB, the time variation of the scaled coefficients b_l^n/b_l^0 is similar to that of the coefficients inverted from surface geomagnetic observations. Our results suggest that it is possible to assimilate geomagnetic observations into numerical dynamo outputs to predict geomagnetic secular variation.

Our results also reveal several problems that we shall work on in the near future for development of a working geomagnetic assimilation system.

First, we shall test the geomagnetic data assimilation with a sequence of insertions. This can force the model outputs closer to the "true states" with the observed geomagnetic secular variation, and provide much better knowledge on error development and model sensitivity to perturbations on, e.g. the torque balances on the Taylor cylinders in the core. Our results show that a strong perturbation on the torque balance could quickly bring the model outputs close to the observations, but destabilize the simulation in a short period. A weaker perturbation may reduce instability problem, however slow down the assimilation. Understanding the error development and model sensitivity will require further studies using observation simulation experiments (OSE's) and Newtonian nudging [Davies and Turner 1977, @] that could eliminate some problems, e.g. non-physical oscillations, inherited from the direct insertion technique. OSE's involve the use of two models (or one model with different parameter values), one of which is treated as the "truth" and the other is the model. Observations taken from the former are then assimilated into the latter.

Another problem to be addressed soon is the appropriate scale between the modeling time and the observation time. The best scaling rules depend on the torsional oscillations which have the frequencies $\sim R_o^{-1/2}$ and decide short-time secular variations [Braginsky 1976, @]. However, with the parameters (10), the torsional oscillations are not separable from other dynamo waves controlled by the leading order magnetostrophic balance in the core [Kuang and Bloxham 1999, @]. Therefore, we need to identify a proper parameter domain in which the torsional oscillations are well separated from other dynamo waves, and computation demand is still manageable.

5. Acknowledgement

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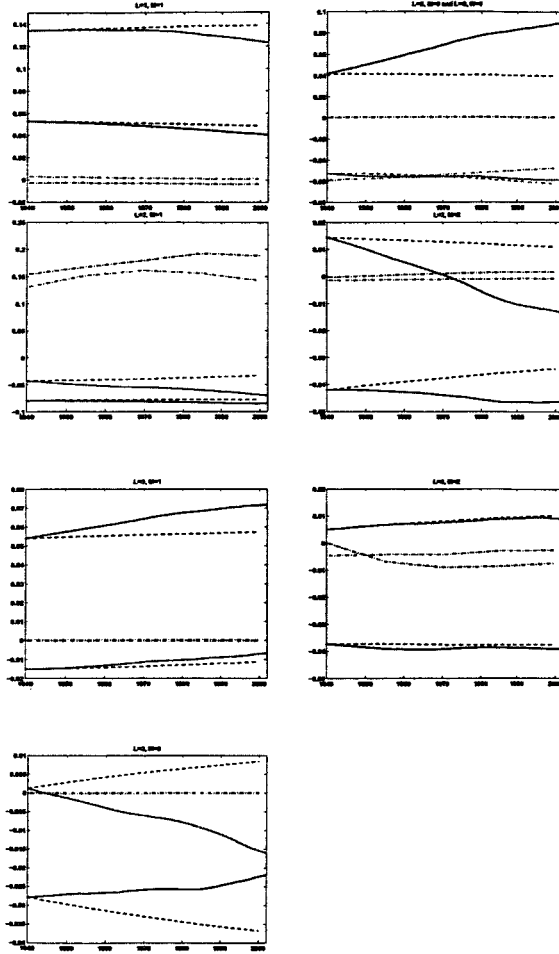


Figure 1. Time variation of the spectral coefficients b_l^m/b_1^0 at the CMB from observations (solid lines), data assimilation (dashed lines) and numerical dynamo modeling (dotted lines). The blue and red lines are the real and imaginary parts of b_l^m , respectively. The green lines are for b_3^0 .

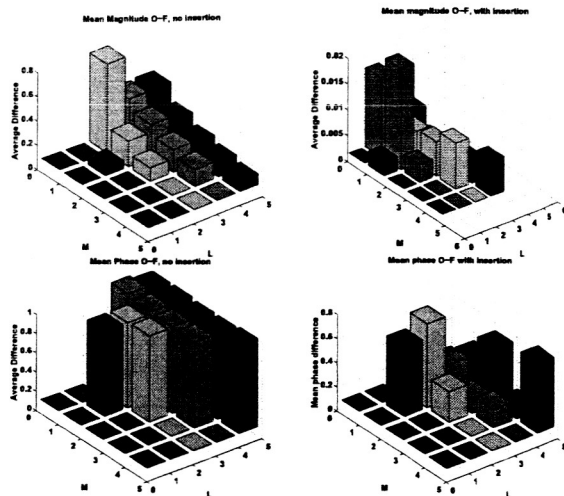


Figure 2. Snapshots of the error analysis of the spectral coefficients b_l^m from observations and the model outputs. The top panel are the errors in the magnitude, and the bottom panel are the errors in the phase. The left panel are the results without insertion, and the right are the results with the insertion (assimilation).

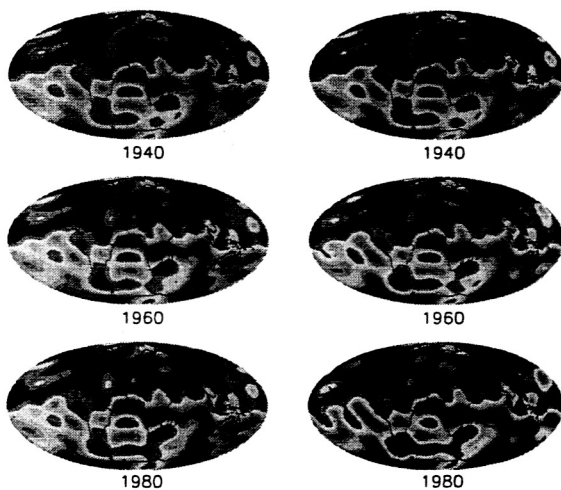


Figure 3. Snapshots of the radial component B_r at the CMB from surface observations (left panel) and from assimilation (right panel).

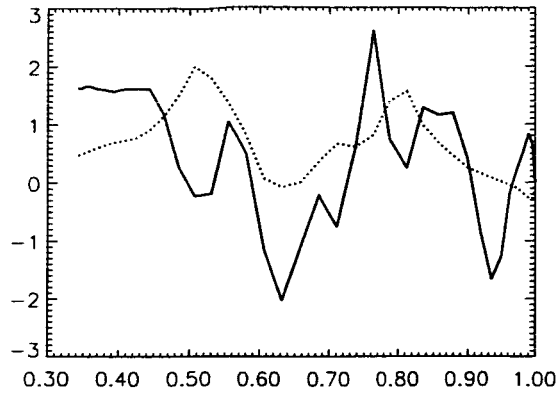


Figure 4. The distributions of the axial torque $\Gamma_z \equiv \int_{\Sigma} (\mathbf{J} \times \mathbf{B})_{\phi} dS$ (\mathbf{J} is the current density and the subscript ϕ denotes the zonal component) on the Taylor cylinders Σ across the the outer core. The dotted line is $50\Gamma_z$ for the dynamo solution, and the solid line is Γ_z modified by the data insertion.